Controller Synthesis for Nonlinear Systems with Time-delay using Model Algorithmic Control (MAC)

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Abstract: A digital controller for nonlinear time-delay system is proposed in this paper. A nonlinear time-delay system is discretized by using Taylor's discretization method. And the discretized system can be converted to a general nonlinear system. For this reason, general nonlinear controller synthesis can be applied to the discretized time-delay system. We adopted MAC controller synthesis for this study. Computer simulations are conducted to verify the performance of the proposed method. The results of simulation show good performance of the proposed controller synthesis and the proposed method is useful to control nonlinear time-delay system easily.

Keywords: MAC control, nonlinear time-delay system, Taylor series expansion

1. INTRODUCTION

Time-delay occurred during the information processing and data transmission in many engineering systems. In recent years, there were many systems that were controlled via network and transferred the data from a remote site owing to the development of networks, in the sense that time-delay, which possibly occurs during the data transmission through the network, is the most important factor for the system performance.

The time-delayed system in the continuous-time space has a property of infinitive dimensions due to time-delay, and the problems of a time-delayed system are not to be able to solve the problems in the continuous-time space. These problems also occurred in the linear time-invariant system where the problems have more complexities and difficulties for solving the problems. For this reason, many control methods developed for the finite dimension system have been experienced in difficulties for applying it directly to the time-delayed system for the last several decades. Therefore, a development of new design methods to control the time-delayed system is required to control it more precisely.

A typical control method for a system, which has time-delay, is a predictive control [1]. A control method using the Smith predictor has been proposed in the fields of process control. This method configures each model with the objective to control the system and time-delay. Thus, it designs a system to remove the effects of time-delay in the characteristic equation of the whole closed loop transfer function through a structural method. Thus, the Smith predictor makes it possible to design a controller by considering a system, which has time-delay, to a system, which doesn't include time-delay. This method has the merit that a controller can be designed using a structural method, regardless of the effects of time-delay, however, it can only be applied to a linear system. In addition, this method has the demerit that an exact model equation for the system and time-delay is required [1][2][8][9].

An estimator is also proposed as an alternative method of predictive control. This estimator calculated state changes in the delayed time using an analysis of the time region of a state equation, and obtains an undelayed and exact plant state for the time that is required to calculate control signals. However, this method only compensates for time-delay between sensors and controllers. However, it is impossible to compensate for the time-delay for the input of the controller. [4][5].

In this paper, we propose a new controller synthesis for nonlinear time-delay system. A nonlinear time-delay system can be discretized using Taylor-series expansion and this discretized system is able to be converted to a general nonlinear system by adopting new internal variables. Thus, the existed nonlinear controller synthesis can be applied to time-delay system. We adopted MAC method for the controller of time-delay system. Model Algorithmic Control (MAC) is a one-step ahead predictive controller, in which the control law is obtained from the minimization of the output error at time $k+r$. It uses an impulse response model to predict the future behavior of the process. MAC was developed in France in the late 70s within the chemical process industry. The original concept was established by Richale et al. (1978), and later the theory was further advanced by Mehra et al. (1980) and Mehar and Rouhani (1980).

This paper consists of the following chapters to explain the results of the study. Chapter 2 presents the method of discretization for a nonlinear system with input time-delay. Chapter 3 introduces the MAC controller synthesis for nonlinear system. Chapter 4 proves the proposed controller for time-delay nonlinear system has a good performance by conducting some computer simulations. And chapter 5 provides the conclusion of this study.

2. TIME-DISCRETIZATION OF NONLINEAR TIME-DELAY SYSTEM

A discrete-time model for a nonlinear continuous-time control system that has time-delay can be obtained using a Taylor-series under the assumption of zero-order hold. This discretization method provides a relatively more exact discrete-time model compared to a continuous-time nonlinear system, and makes it possible to apply the existing nonlinear control method to a discrete system, which includes time-delay.

A continuous-time nonlinear control system, which has a single input, can be presented as Eq. (1) using a state-space expression.

$$\frac{dx(t)}{dt} = f(x(t)) + g(x(t))u(t-D) \tag{1}$$

where $x \in X \subset R^n$ represents the system state, $u \in R$ is an input variable, D is time-delay, and $f(x)$ and $g(x)$ are nonlinear functions for x, respectively. In addition, the zero-order hold was assumed for a fixed sampling period, and constant input in a single sampling region.

$$\begin{aligned} &u(t) = u(kT) \equiv u(k) = \text{constant}, \quad kT \le t < kT + T \\ &D = qT + \gamma \end{aligned} \tag{2}$$

where T is sampling phase, q is an integer multiple of $q \in \{1, 2, 3, \ldots\}$ for the sampling period, γ is a small time-delay of $0 < \gamma \le T$. The delayed input variable was applied to the system that has values for the different sampling regions, as presented in Eq. (3).

$$u(t-D) = \begin{cases} u(k-q-1) & \text{if } kT \le t < kT + \gamma \\ u(k-q) & \text{if } kT + \gamma \le t < kT + T \end{cases} \tag{3}$$

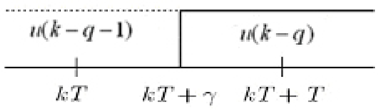

Fig. 1 Delayed input signal

A discrete system for the nonlinear system that has input time-delay can be configured as Eq. (4).

$$\begin{aligned} x(k+1) &= x(k) + \sum_{l=1}^{M} A^l(x(k), u(k-q-1))\frac{\gamma^l}{l!} \\ &+ \sum_{l=1}^{M} A^l((x(k) + \sum_{i=1}^{M} A^i(x(k), u(k-q-1))\frac{\gamma^i}{i!}), u(k-q))\frac{(T-\gamma)^l}{l!}, \end{aligned} \tag{4}$$

where $x(k)$ is the value of a state vector of x at $t = t_k = kT$, M is truncation order of the Taylor-series. $A^{[l]}(x,u)$ is cyclically defined by Eq. (5).

$$\begin{aligned} &A^{[1]}(x,u) = f(x) + ug(x) \\ &A^{[l+1]}(x,u) = \frac{\partial A^{[l]}(x,u)}{\partial x}(f(x) + ug(x)). \end{aligned} \tag{5}$$

The discrete expression for Eq. (1), which is the original continuous-time systems, is presented by Eq. (6).

$$x(k+1) = \Phi_T^M(x(k), u(k-q-1), u(k-q)) \tag{6}$$

where the function Φ_T^M depends on the sampling period of T and truncation order of M. As mentioned above, the discretization of a nonlinear system using a Taylor-series presented better results than that of the existing Euler method. The comparison can be performed using discretization errors.

In order to bring system (6) to the standard state-space sampled-data representation form of the original continuous-time system where the input $u(k)$ at time kT explicitly appears in the dynamic equations, let us now define auxiliary state variables for the past input values as follows:

$$\begin{aligned} z_1(k) &= u(k-q-1) \\ z_2(k) &= u(k-q) \\ &\vdots \\ z_q(k) &= u(k-2) \\ z_{q+1}(k) &= u(k-1) \end{aligned} \tag{7}$$

whose dynamic can be realized by the following difference equations:

$$\begin{aligned} z_1(k+1) &= z_2(k) \\ z_2(k+1) &= z_3(k) \\ &\vdots \\ z_q(k+1) &= z_{q+1}(k) \\ z_{q+1}(k+1) &= u(k) \end{aligned} \tag{8}$$

Therefore, the standard sampled-data representation of (1) in state-space is given by the following augmented nonlinear discrete-time system:

$$\begin{bmatrix} x(k+1) \\ z_1(k+1) \\ . \\ . \\ . \\ z_{q+1}(k+1) \end{bmatrix} = \begin{bmatrix} \Phi_T^D(x(k), z_1(k), z_2(k) \\ z_2(k) \\ . \\ . \\ . \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 1 \end{bmatrix} u(k) \tag{9}$$

Denoting $[x, z_1, \ldots, z_{q+1}] = \bar{x}, \quad \Phi_T^D(\bar{x}) = \begin{bmatrix} \Phi_T^D(x(k), z_1(k), z_2(k)) \\ z_2(k) \\ . \\ . \\ . \\ 1 \end{bmatrix},$ and $B = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 1 \end{bmatrix}$, the above sampled-data representation can be written in a more compact form:

$$\bar{x}(k+1) = \Phi_T^D(\bar{x}(k)) + Bu(k) \tag{10}$$

3. Model Algorithmic Control(MAC) for nonlinear system

3.1 State-space formulation of MAC for linear processes

Consider linear processes described by a discrete-time state space model of the form of Eq. (11).

$$\begin{aligned} x_m(k+1) &= Ax_m(k) + Bu(k) \\ y_m(k) &= Cx_m(k) \end{aligned} \tag{11}$$

where the subscript *m* has been added to explicitly indicate that x_m and y_m represent estimates of *x* and *y* obtained by simulating the model, given the manipulated input *u(k)*. This notation will help differentiate y_m from the measured output, which will still be denoted by *y*. The model of Eq. (11) can be simulated on-line to predict the future behavior of the process. In particular, from the model described by Eq. (11), we obtain:

$$\begin{aligned} y_m(k) &= Cx_m(k) \\ y_m(k+1) &= CAx_m(k) \\ &\vdots \\ y_m(k+r-1) &= CA^{r-1}x_m(k) \\ y_m(k+r) &= CA^r x_m(k) + CA^{r-1}Bu(k) \end{aligned} \qquad (12)$$

from which one can predict the future changes in the output as follows:

$$\begin{aligned} y_m(k+1) - y_m(k) &= (CA-C)x_m(k) \\ y_m(k+2) - y_m(k) &= (CA^2-C)x_m(k) \\ &\vdots \\ y_m(k+r-1) - y_m(k) &= (CA^{r-1}-C)x_m(k) \\ y_m(k+r) - y_m(k) &= (CA^r-C)x_m(k) + CA^{r-1}Bu(k) \end{aligned} \qquad (13)$$

When these predicted changes are added to the measured output signal *y(k)*, one obtains future predictions of the output:

$$\begin{aligned} \hat{y}(k+1) &= y(k) + (CA-C)x_m(k) \\ \hat{y}(k+2) &= y(k) + (CA^2-C)x_m(k) \\ &\vdots \\ \hat{y}(k+r-1) &= y(k) + (CA^{r-1}-C)x_m(k) \\ \hat{y}(k+r) &= y(k) + (CA^r-C)x_m(k) + CA^{r-1}Bu(k) \end{aligned} \qquad (14)$$

where the superscript ^ is used to indicate that $\hat{y}$ represent a prediction of the output. It is interesting to observe that the output predictions in Eq. (14) are "closed-loop" predictions in the sense that they make use of the measured output signal. Also that the manipulated input move *u(k)* affects the output after *r* sampling periods, and this conforms with the interpretation of $r\Delta t$ as the overall delay of the system [$(r-1)\Delta t$ is the process dead time, and Δt is the sampling delay].

At every time step, the control computer can calculate the output prediction Eq. (14), driven by *u(k)* and *y(k)*, where $x_m(k)$ is obtained by on-line simulation of the state equations of Eq.(11):

$$x_m(k+1) = Ax_m(k) + Bu(k)$$

The question that arises then is what should be the choice of *u(k)* to obtain a desirable output response after *r* time steps. If *u(k)* is chosen so that $\hat{y}(k+r)$ is exactly the set-point value, this would clearly create a nonrobust situation. Instead, one can request $\hat{y}(k+r)$ to be in the right direction and cover a fraction of the "distance" between $\hat{y}(k+r-1)$ and the set-point value. In other words, one can define a desirable value of the output at the (*k*+*r*)th time step by:

$$y_d(k+r) = (1-\alpha)y_{sp} + \alpha\hat{y}(k+r-1) \qquad (15)$$

where α is a tunable filter parameter such that $0<\alpha<1$. Clearly, $\alpha \to 0$ corresponds to $y_d(k+r) \to y_{sp}$ and, therefore, will try to force the output to go to set point as soon as possible, whereas $\alpha \to 1$ corresponds to $y_d(k+r) \to \hat{y}(k+r-1)$, leaving the output unaffected. An intermediate choice of α corresponds to a desirable value of the output in between y_{sp} and $\hat{y}(k+r-1)$ that tries to bridge the gap to a certain extent. Equation (15) is referred to as the "reference trajectory" in the MAC literature.

Once the reference trajectory has been specified, the question then becomes how to choose the control move *u(k)* so that $\hat{y}(k+r)$ will match $y_d(k+r)$. this can be formulated as an optimization problem:

$$\min_{u(k)} [y_d(k+r) - \hat{y}(k+r)]^2 \qquad (16)$$

or, equivalently, in view of Eqs. (14) and (15):

$$\begin{aligned} \min_{u(k)} [(1-\alpha)e(k) - [(CA^r - C) \\ -\alpha(CA^{r-1} - C)]x_m(k) - CA^{r-1}Bu(k)]^2 \end{aligned} \qquad (17)$$

3.2 Nonlinear MAC for nonlinear processes

The steps of the state-space linear MAC of the previous subsection can be extended "word by word" to nonlinear processes described by discrete-time models:

$$\begin{aligned} x_m(k+1) &= \Phi[x_m(k), u(k)] \\ y_m(k) &= h[x_m(k)] \end{aligned} \qquad (18)$$

where again the subscript *m* is added to indicate estimates of *x* and *y* obtained by model simulation and differentiate the simulated *y* from the measured *y*.

On-line simulation of the model described by Eq. (18) can be used to predict the future changes in the output *y* as follows:

$$\begin{aligned} y_m(k+1) - y_m(k) &= h^1[x_m(k)] - h[x_m(k)] \\ y_m(k+2) - y_m(k) &= h^2[x_m(k)] - h[x_m(k)] \\ &\vdots \\ y_m(k+r-1) - y_m(k) &= h^{r-1}[x_m(k)] - h[x_m(k)] \\ y_m(k+r) - y_m(k) &= h^{r-1}[\Phi[x_m(k), u(k)]\} - h[x_m(k)] \end{aligned} \qquad (19)$$

When these predicted changes are added to the measured output signal *y(k)*, one obtains the following "closed-loop" predictions of the output:

$$\begin{aligned} \hat{y}(k+1) &= y(k) + h^1[x_m(k)] - h[x_m(k)] \\ \hat{y}(k+2) &= y(k) + h^2[x_m(k)] - h[x_m(k)] \\ &\vdots \\ \hat{y}(k+r-1) &= y(k) + h^{r-1}[x_m(k)] - h[x_m(k)] \\ \hat{y}(k+r) &= y(k) + h^{r-1}[\Phi[x_m(k), u(k)]] - h[x_m(k)] \end{aligned} \qquad (20)$$

Defining a linear reference trajectory as same as in the linear case (Eq. (15)):

$$y_d(k+r) = (1-\alpha)y_{sp} + \alpha\hat{y}(k+r-1)$$

one can derive a nonlinear MAC controller by requesting the output prediction t match the reference trajectory in the sense of minimizing the performance index of Eq. (16):

$$\min_{u(k)} [y_d(k+r) - \hat{y}(k+r)]^2$$

In view of Eqs. (15) and (20), this becomes:

$$\begin{aligned} \min_{u(k)} \{(1-\alpha)e(k) - h^{r-1}\{\Phi[x_m(k), u(k)]\} + \\ \alpha h^{r-1}[x_m(k)] + (1-\alpha)h[x_m(k)]\}^2 \end{aligned} \qquad (21)$$

In the absence of the input constraints, this minimization problem is trivially solvable. Minimizing *u(k)* is the solution of the nonlinear algebraic equation:

$$h^{r-1}\{\Phi[x_m(k),u(k)]\} = \alpha h^{r-1}[x_m(k)] + (1-\alpha)h[x_m(k)] + (1-\alpha)e(k) \quad (22)$$

4. COMPUTER SIMULATIONS

In this section, two computer simulations are conducted to show the performance of proposed controller. One is CSTR model and the other is Van der Pol system.

First, CSTR model is conducted. CSTR is a general nonlinear chemical process and the equation of the model can be expressed as shown in Eq. (23):

$$x'(t) = -x^2(t) - 3x(t) + u(t - D(t))(1 - x(t)) \quad (23)$$

Using Taylor-series expansion, the above system can be discreitzed as representation forms of following:

$$\bar{x}(k+1) = \Phi_T^D(\bar{x}(k)) + Bu(k)$$

The on-line model for MAC controller is presented by the truncation order of *N=2* of Taylor' s discretization method. Initial value is assumed $x(0) = 0$. Time-delay which is exerted on input is assumed *D=1*. α at reference trajectory is assumed 0.7 and the set-point value is 0.7. Fig. 1 shows the response of MAC controller of CSTR system.

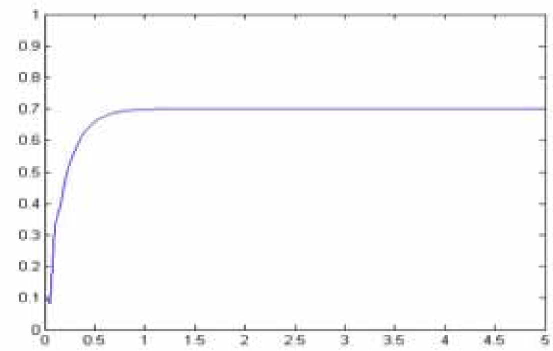

Fig. 1 Response of CSTR under MAC controller

Second, Van der Pol system is conducted. Van der Pol system is a typical nonlinear system. This system can be analyzed using a mass-spring-damper system, which has a position-dependent damping coefficient, and a RLC electric circuit. If this system has an initial value besides an equilibrium point, a periodical vibration will be maintained in a limited region. This periodical vibration is called a limit cycle. The system can be expressed using a dynamics equation as presented in Eq. (24). The state space expression is Eq. (25).

$$x'' = x'(1 - x^2) - x + u \qquad y = x \quad (24)$$

$$X_1' = f_1(X) + g_1(X) = X_2 \qquad X_2' = f_2(X) + g_2(X)u = X_2(1 - X_1^{\,2}) - X_1 + u \quad (25)$$

where the state vector is $X = [X_1 \quad X_2]^T = [x \quad x']^T$. In the case of the existing input time-delay, such as $D = qT + \gamma$, the discrete expression is expressed as Eq. (26) using the Taylor's discretization method.

$$X_1(k+1) = X_1(k) + \sum_{l=1}^{M} A_1^{\,l}(X(k),u(k-q-1))\frac{\gamma^l}{l!} + \sum_{l=1}^{M} A_1^{\,l}((X(k) + \sum_{l=1}^{M} A_1^{\,l}(X(k),u(k-q-1))\frac{\gamma^l}{l!}),u(k-q))\frac{(T-\gamma)^l}{l!}$$

$$X_2(k+1) = X_2(k) + \sum_{l=1}^{M} A_2^{\,l}(X(k),u(k-q-1))\frac{\gamma^l}{l!} + \sum_{l=1}^{M} A_2^{\,l}((X(k) + \sum_{j=1}^{M} A_1^{\,j}(X(k),u(k-q-1))\frac{\gamma^j}{l!}),u(k-q))\frac{(T-\gamma)^l}{l!} \quad (26)$$

where the item of the partial differentiation of $A^l(x,u)$ is to be cyclically defined as follows.

$$A_1^{\,l}(x,u) = f_1(X) + g_1(X)u$$

$$A_1^{\,l+1}(X,u) = \frac{\partial A_1^{\,l}(X,u)}{\partial X_1} f_1 + \frac{\partial A_1^{\,l}(X,u)}{\partial X_2} f_2$$

$$A_2^{\,l}(X,u) = f_2(X) + g_2(X)u$$

$$A_2^{\,l+1}(X,u) = \frac{\partial A_2^{\,l}(X,u)}{\partial X_1} f_1 + \frac{\partial A_2^{\,l}(X,u)}{\partial X_2} f_2 \quad (27)$$

The on-line model for MAC controller is presented by the order of *N=2* of Taylor's discretization method. Initial values of the system are $x_{(0,0)} = [0.1 \quad 0]^T$. Adjustable parameter of reference trajectory is $\alpha = 0.7$ and set point is set by 1. Fig. 2 shows the output of MAC controller of Van der Pol system.

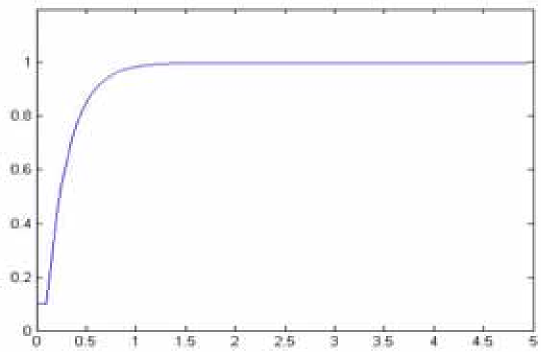

Fig. 2 Response of Van der Pol system under MAC controller

5. CONCLUSIONS

In this paper, we proposed a new controller synthesis for nonlinear time-delay system. Taylor series expansion is used to obtain a discrete-time nonlinear system of continuous-time system, and the discretized system can be converted to a general nonlinear system by using auxiliary variables. A digital controller is designed on this system using MAC controller synthesis. Two computer simulations are conducted to verify the performance of the proposed method and the results of simulations show good performance and useful.

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